

TIME AND EVOLUTION

A.M. MOLCHANOV

Introduction.

Someone (an Englishman, if one were to judge by the style) once said: "When God created time, he created enough of it". Another clever Englishman, Charles Darwin, describes his feelings, evoked by the geological scale of time, in just as clear a fashion, although, perhaps, more emotionally: "...the mind is stupefied in thinking over the long, absolutely necessary, lapse of years" (quoted from Collier edition, New York, 1909, p.185). Another passage of his is no less expressive: "...yet we must confess that it makes the head almost giddy to reflect on the years, century after century" (loco cit., p. 196).

It is edifying to qualitatively imagine the scale of time, which evoked such emotional upheaval in Darwin. Both passages touch upon the amount of time it took for the formation of the valley of the Santa Cruz River in Patagonia. Darwin writes that it happened "much after the formation of the cushion course with tertiary shells". In fact, one would now estimate this period of time to have taken several dozens of millions of years, which is a modest stretch of time, compared to the age of the Earth, which is at least a hundredfold older.

But what has agitated Darwin so much? And why do we talk so calmly of much larger periods of time after only one century? And what is better - Darwin's agitation or our tranquility?

The last question needs not be answered due to its rhetorical nature. But the other two could be of interest.

Catastrophe or evolution?

The incessant tension of the fight against catastrophism penetrates Darwin's entire book. One has an easy way of dealing with the theory of catastrophes in modern popular (and not very) scientific literature - it is branded as unscientific. However, the real Darwin, and especially his friend and predecessor Lyell had much more difficulty with it, since the age of the Earth at that time was unknown.

Interestingly, Darwin, a man, almost pedantic in everything that has to do with size, amount and even prices (page 383 alone contains over a dozen different numbers) becomes descriptive, picturesque and emotional as soon as he touches upon the estimation of time. The only time he talks about it qualitatively - "like unto a geologist who had lived his ten thousand years" - is a metaphor, but one that makes the reader think. Was Darwin's qualitative reference point the Biblical seven thousand years? If this is indeed the case, many things become clear. It is clear, for instance, why Newton, who lived one hundred and fifty years prior to Darwin, was forced to consider highly symmetrical structure of the Solar system as a proof of God's hand in its creation - there was simply no time for it to

have natural origin. This is the origin of Newton's "first push of God" that got him his share of scolding from Engels later on.

Cuvier's catastrophies seem to stem from the same source. One just could not fit into the boundaries set by naturalists on the natural history, whether it was done consciously or not.

Two solutions were logically justified - either a series of catastrophes (which was also suggested by the biblical turmoil of the first six days), or a radical revision of the question of time. As we now know, for that one would need to extend the history of Earth million fold.

Procrustes misdemeanor is mere dilettantism compared to what science would have to go through. Thus it is quite clear why Charles Lyell and Charles Darwin - the people who did more than others for consolidation of the idea of evolution - tried not to risk much with statements that were qualitative in nature. Data, primarily geological, that was gathered and analyzed by them, forced the conclusion about vastness of the time that has passed. However, any qualitative methods could only appear only after and as a result of the psychological breakthrough that they were creating.

Big numbers. "Why are atoms so small?" - asked Erwin Schrödinger in the book "What is life from the point of view of physics". He answered his own question - "It's because we are so big". A complex system (a person) inevitably contains an enormous number of simple atoms - that's the main idea behind Schrödinger's statement. However, there is a flip side to the "person - atom" comparison. One cannot just say "a lot" or "a little", irrelatively. Any qualitative statement is a statement about objects of the same type. Mathematically speaking, parameters need to be dimensionless. Thus, Schrödinger emphasizes that every small parameter is a flip side of a big parameter. However, it is the "form" of the question that is psychologically interesting. Moreover, it has substantial methodological meaning. "Why we are big" is a deeper question because it immediately gives rise to another question: could we be smaller? And if so, how much smaller? And it turns out that a seemingly harmless paraphrase changes the problem at the core. Mere curiosity leads to a fundamental problem: what are the minimal, critical values of volume, mass, energy, time, etc., that are sufficient for the existence of life?

Darwin is really quite melancholic when he talks about the "sequence of centuries". It is just a concealed sigh about how short a human life is and a respectful astonishment at the age of even such a geologically ordinary object as a river valley. But what does a man with his allotted time have to do with this? It is sad, of course, that his time is so short (compared to what?). But a man just stands there, looking and trying to understand. After all, it was not he who created the valley but the daily ebb and flow of the sea sleeve that later on became the Santa Cruz River. The scale of human life here is really just an external influence, and even the astronomical value of time is significant only in its relation to the "singular act of creation".

It would be quite instructive to leave the two main characters, basalt massif and tidal wave, alone for a while. The massif is a good million years old, and every day of every year of these million years the tidal wave "throws itself on the rocks and, in white foam, runs away". A billion times.

It did not take much effort for us to ease into saying “ten to the ninth power”, and even less effort to forget how to be astonished at the magnitude of it. Darwin could still do it, and pretty well at that.

To see eternity in an hour. The largest time segment that can more or less be estimated is 10^{13} , the age of our galaxy. A mistake in one or two orders of magnitude here really makes no difference.

The smallest time segment that can be discussed at modern level of knowledge is 10^{-23} . It is the time that light would need to travel from “one side of an electron to the other”, if such a phrase made any sense. Luckily, for our purposes a time estimate that comes from dividing the radius of an electron (10^{-13}) by the speed of light (3×10^{10}) will suffice. What interpretation this “time quantum” will gain in the future elementary particle theory is not significant as of now.

The only solid link in the midst of these two numbers is the segment that connects them - the number of seconds in a year. And even that is questionable - it is quite possible that the length of a day, and of a year for that matter, changes. However, from 1969 data we can calculate it to at least two significant digits: $365 \times 24 \times 60 = 0.32 \times 10^8$.

One should compare these barely comparable numbers in logarithmic scale. And it's not just a matter of convenience - as we will see, such a scale in fact has very deep structural meaning.

All “Eternity” turns out to be not that long - just 44 orders of magnitude. Out of those cosmos (from the age of the Galaxy to a year) gets 13 orders of magnitude, 8 (from year to a second) are given to people, and the remaining 23 - over a half - go the kingdom of microorganisms. Poet W. Blake invites us “to see Eternity in an hour”. However, one Hour contains an enormous portion of time (28 orders of magnitude out of 44), so Blake's poetic intuition turns out to be quite a rough instrument for these purposes. Naturalist C. Darwin guesses Eternity much sooner (to nine orders of magnitude) and feels it much better - not just in length or flow of time but in the creating rhythm of the tide. And if anybody tends toward seeing tide as a destructive force rather than a creative one (although, creation does always come at the expense of destruction), they should go and read Darwin's theory of the creation of atolls, where the creative nature of the tide is pretty much unquestionable. Of course, in this form “calculation of Eternity” is somewhat comic in nature; however, it gives a general sense of what the scale of time is like. It is possible that this scale will change somewhat at extremes - on the cosmological “top” and quantum “bottom” - but the entire biological evolution must remain “between” the two extremes. Thus, for all the complexity in the creation of life there are about 40 orders of magnitude - is that enough? Or there will once again be a need for catastrophes? That's the not so comic meaning of this discussion. Interestingly, Blake's “eternity” is close to about half of the Eternity. A moment, an instant, an eye blink, lasts just a fraction of a second. Our Galaxy has seen as many eye blinks during the time of its existence, as there are time “quanta” in one eye blink. Man is a macrocosm, a “measure of all things”, and qualitatively he is right in the middle of the micro-world and cosmos.

Mathematical Intermezzo. In mathematics, in non-linear theory of oscillations, there exists a problem that is quite similar to the one that we have been looking at all this time.

Consider the following system of equations:

$$(1) \quad \begin{aligned} \frac{dx}{dt} &= f(x, y) \\ \frac{dy}{dt} &= \epsilon g(x, y) \end{aligned}$$

which contains a small parameter ϵ . What is the meaning of this parameter? It can be interpreted (although - and that is very important - the interpretation is not unique) as a relation between the time scales of the “ x ” system and the “ y ” system.

One can approximate the change in x as

$$(2) \quad \Delta x \approx f \Delta t$$

From this we can get a time estimate of

$$(3) \quad \Delta t \approx \frac{|\Delta x|}{|f|},$$

which is crucial for the x -system to experience the changes in the scale of Δx .

Therefore, the scale of the time of “significant” changes in the dynamics of the system $\Delta x \approx 1$ is on the order of 1. We assume, of course, that the scales of x, y and t were chosen appropriately, such that the values of $f(x, y)$ and $g(x, y)$ would be of order 1. A similar argument for the variable y leads to the following result:

$$(4) \quad \Delta t \approx \frac{1}{\epsilon} \frac{|\Delta y|}{|g|}$$

which means that an enormous amount of time

$$(5) \quad T \sim \frac{1}{\epsilon}$$

is necessary to see any significant changes, and it gets larger and larger as ϵ gets smaller. Just this circumstance alone suggests that ϵ characterizes (and that’s another interpretation) the lack of real interaction between the systems x and y .

This elementary analysis shows something very important: the system that is described by vector x changes nearly independently from the system that is described by vector y . In this case one usually calls x the “fast” variable and y the “slow” variable. A convenient formal trick is to study the limit by setting $\epsilon = 0$.

Of course, one cannot ignore the fact that the core of mathematics lies in studying asymptotic, idealized, extreme situations. Situations that are simple enough to be analyzed logically but complex enough to preserve the main characteristics of the situation that is being described; otherwise, the analysis is useless. Graphics, or even caricature on the phenomenon, or situation is mathematics’ main tool - its main target is to capture the “defining” in the object. Hence, watercolor semitones have no place in the mathematical approach, at least at the stage of posing the question.

But back to our problem. We obtain the following system of equations:

$$(6) \quad \begin{aligned} \frac{dy}{dt} &= 0, \\ \frac{dx}{dt} &= f(x, y) \end{aligned}$$

from which we can see that

$$(7) \quad y = y_0 = \text{const.},$$

so the only equation that remains is

$$(8) \quad \frac{dx}{dt} = f(x, y_0).$$

The system for x thus depends only on the current value of y_0 and not on the dynamics of the variable y . Fast variables change and evolve, while slow variable remain nearly constant. In a similar mindset we talk without thinking about the height of a mountain above the sea level even if we find sea shells on top of it, which indicate that the top of the mountain was actually once the bottom of the sea.

It is unlikely that anyone who is reading this noticed (even professional mathematicians rarely notice it) that this entire discussion is very unequal with respect to slow and fast variables.

We were obviously subjective. It is formally evident in the fact that time is measured on the scale of fast variables. It is the “eigen-time” of the fast variables that is taken to be the time unit, whether it be the oscillatory period in case of a periodic function, or half-life in case of functions that describe decay.

Let’s try to restore justice. Consider the “slow” time

$$(9) \quad \tau = \epsilon t.$$

In this new time the system becomes

$$(10) \quad \begin{aligned} \epsilon \frac{dx}{dt} &= f(x, y), \\ \frac{dy}{dt} &= g(x, y), \end{aligned}$$

and limit transition (whose full analysis still poses a difficult problem that has not yet been entirely solved) leads to the conclusion:

$$(11) \quad \begin{aligned} 0 &= f(x, y) \\ \frac{dy}{dt} &= g(x, y). \end{aligned}$$

Once again we are dealing with just one system, but this time it is of course the system of slow variables. Fast variables, like slow variables in the previous case, are now effectively eliminated from the system. We can thus make a curious general conclusion. Fixing a

particular time scale “turns off” or “freezes” other time scales. Interestingly, it is not only the slow processes that “freeze” (which is quite understandable) but also the fast ones, which at first seems quite paradoxical.

However, it is so, and the following example is to give a vivid illustration of this. Let’s assume that we are looking at a propeller from a plane and at a second hand in a watch that move simultaneously. One can clearly see the second hand in the watch moving - it is the “main” variable. The minute and hour hands are “frozen” in one place - that is the “slow” variable. However, in case of the plane, we do not see the propeller moving - we just see a *motionless, stationary* circle. In this case a mathematician would speak of “averaging by the trajectory of the fast movement”.

In this case it is not about optical illusions or psychology. For instance, an atom is “transparent” to α -particles but is impermeable to other atoms. One can get a good classical model of this phenomenon by spinning a bicycle wheel very quickly. Sticking your finger there might hurt, but small stones thrown at it will easily pass through the wheel that is “impermeable” to slower motions.

Is it only the micro world that qualifies for a quantization? Of course, “freezing” of the fast processes is qualitatively different from the “freezing” of the slow processes, however similar they may seem at first.

The difference lies in the fact that we can “catch” the slow processes at any stage of their development while the fast processes always manage to evolve and reach a steady state before we can get to them.

In the case of our systems, slow variables y_0 in System (6) can take on any values, and so we subjectively perceive this as continuity. However, fast variables x must satisfy the first equation in System (11), i.e. reach a quasi-steady state. The word “quasi” in this case is used to identify the dependence on the value of the slow variable and especially to emphasize the fact that equilibrium could be lost if y were to keep evolving. Hence, the sequence of possible states of the fast variables x becomes discrete.

From this point of view the fact that it is in the micro world that discontinuity was first understood is only natural. However, it is only a specific example of a much more general trend.

The “final” states (quasi-stable, meta-stable, stable) are clearly discrete. That’s why evolutionarily mature systems always tend towards discrete hierarchical structure. Biological systems are no exception. The “mush” of continuity is as contra-indicative for them as it is for quantum objects, and for the same reason - because of evolutionary maturity.

The role of dissipation. When Faustus asks Mephistopheles who he is, the latter answers quite elusively: Ein Teil von jener Kraft, die stets das Böse will, und stets das Gute schafft (I am part of force that always wants the evil and always does the good). Of course, one can’t trust the Devil: he’ll lie, will not charge much and is always happy to cheat a “good Christian”. However, in this case he actually said the truth, although not in its entirety.

The modern scientific version (and yet another form that Mephistopheles chose to take) is dissipative factors: friction, inelastic crash, age of the material, atmospheric escape, accumulation of mutations, etc. All of them seem to be the entropy-increasing devil’s minions. Indeed, if these factors are strong enough, they will do their job and cause the

system to degrade completely. For instance, if the Earth experienced enough friction while moving along its orbit, it would eventually fall on the sun and burn together with its entire biosphere.

However, if these factors are kept under control, instead of the Devil we end up with a legion of quite useful little imps that are doing something that is actually quite useful. It may seem that the impressionism of the past few paragraphs is just an attempt to replace, even if only on the emotional level, numerous preceding pages of tiresome mathematical text - that under certain conditions dissipative factors (particularly their “expenditure”, their decrease over the course of evolution) have a tendency to stabilize complicated oscillating structures.

If the dissipation is large and the number of discrete resonance levels is small, the structure turns out to be quite dull and uninteresting. That is of course quite true. However, one cannot have no dissipation at all - that causes the system to become “mushy”, stabilizing factor disappears and the system is then perceived as purely stochastic, and in fact becomes equivalent to it in the statistical sense of the word.

Time hierarchy. Previously we took a brief look at the interactions between just two systems with vastly different time scales. In reality the chain of time scales that are “tacked” into each other is much longer.

For instance, one can take the time necessary for one catalytic reaction as an initial time scale when studying a biological system (a mammal, for example). For the fastest enzymes it would be on the order of 10^{-4} or even 10^{-6} seconds. One of the results of a long sequence of such events could be muscular contraction, which lasts about 10^{-2} seconds. The next scale that we can take could be one heart beat - one second. Time, necessary for a relatively complicated muscular act, such as a conditional reflex, would be measured in minutes. Digestive reflex is measured in hours. And so on. However, the analysis given above demonstrates that looking at the entire hierarchy of time scales and corresponding systems is not necessary, at least at first. It is the “freezing” of the lowest or highest time scales that is of primary methodological importance. It allows study of just two neighboring time scales, which is a very important step - identifying the “quantum”, or structural element of evolution.

Therefore the problem divides into two problems that are very different in character and style.

The study of interactions of two systems in neighboring time scales inevitably assumes subsequent study and classification of the possible dynamic regimes. We can immediately see that the process on each level can be of at least three types: stabilizing (sink), destabilizing (source) and oscillatory. Since there are at least two levels, the number of possible behaviors increases to a minimum of nine. In reality there could be many more than that, since, for instance, oscillatory regimes can be periodic (single-frequency) or multi-frequency, each with its own qualitative characteristics. Description of the entire “chain”, however, is quite different in character - it’s systematic, classifying, or one can say that even cybernetic in nature. In this case creating a “list” of connections between different chains would suffice, and kinetic issues are as good as pre-packaged in the type of the connection.

It is very important to note that a system that reaches equilibrium on some particular level may turn out to be oscillatory or even unstable on a large time scale. There is no real connection between the types of behavior on different levels. For instance, gas that has reached equilibrium and is in accordance with the second law of thermodynamics can very well be a working force in a steam engine. There is no contradiction here since the time necessary for gas relaxation is a fraction of a millisecond, while the mechanic work in the engine is done on the scale of seconds.

Kinetics and structure. The problem of how the system behaves in time is of great methodological importance, which is in-built in the very nature of the theory of evolution. If everything were not created in its final state and form, if everything was developing gradually, then kinetics of each system must have preceded its structure. And in evolutionary sense it is undoubtedly so. However, it is much more curious that recreation of a system that has once already been created follows the same steps as did the very first system.

That is the general meaning of the well-known thesis about repetitions in individual development of everything that is evolutionary in origin.

However, this repetition can't be retracing every single step. Otherwise, every birth of every human being would once again take three billion years of evolution. And so an enormous, mind-blowing time reduction and step simplification has to take place. And it is the understanding of the general principles of this acceleration without loss of the obtained structural level that is one of the most important theoretical problems that we are dealing with. A problem that is not only theoretical but perhaps also practical, even technological in nature.

[Translated by I. Kareva (ikareva@asu.edu)]